Online Appendix for Effort, Talent, and Inequality in a Small Open Economy*

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A Additional Derivations and Results

A.1 Optimality of linear contracts

Linear performance-pay contracts are optimal under bilateral risk neutrality and achieve the first-best solution to the firm's problem. This result complements the findings of Holmstrom and Milgrom (1987), establishing the linearity of optimal intertemporal compensation for an agent with constant absolute risk aversion.[1]

Suppose entrepreneurs and workers have the same information, with each entrepreneur observing worker effort e_i but not ξ_i. The entrepreneur observes aggregate worker output $\int_0^l x_i di$ because parameters μ and λ are public knowledge. Aware of the symmetry of production technology across workers, each entrepreneur offers the same fixed wage w to all workers and maximizes the expected firm profit subject to each worker's participation constraint.

The worker participation constraint implies that each worker sets $V_0 = (\tau p)^{-\theta}(r + w_0)$ equal to $E[V_i] = (\tau p)^{-\theta}[r + w] - ce_i^2/2$ and yields

$$E(V_0) = E[V_i] \qquad \Rightarrow \qquad w = w_0 + c(\tau p)^{\theta}\frac{e_i^2}{2}, \tag{A1}$$

where e_i is observable effort. Equation (A1) implies that, by offering a wage w, the entrepreneur requires the same effort $e_i = e$ from each worker.

Firm profit is

$$\begin{aligned} \pi_z &= \tau p\gamma(a + \lambda e + \mu)z^{1/2}l^{1/2} - \int_0^l w di \\ &= \tau p\gamma(a + \lambda e + \mu)z^{1/2}l^{1/2} - l\left(w_0 + c(\tau p)^{\theta}\frac{e^2}{2}\right). \end{aligned} \tag{A2}$$

Maximizing profit π_z with respect to effort e and the measure of workers l delivers

$$\frac{\partial \pi_z}{\partial e} = 0 \qquad \Rightarrow e = \frac{\lambda\tau p\gamma}{c(\tau p)^{\theta}}\left(\frac{z}{l}\right)^{1/2}, \tag{A3}$$

and

$$\frac{\partial \pi_z}{\partial l} = 0 \qquad \Rightarrow \qquad \frac{1}{2}\tau p\gamma[a + \mu + \lambda e]\left(\frac{z}{l}\right)^{1/2} = w_0 + c(\tau p)^{\theta}\frac{e^2}{2}. \tag{A4}$$

[1] Edmans and Gabaix (2016) provide an excellent and intuitive analysis of the optimality of linear contracts under risk neutrality and unlimited liability in the context of executive compensation. In their case, the principal is the firm's board of directors and the agent is the CEO.

Substitute $\left(\frac{z}{l}\right)^{1/2} = ec(\tau p)^{\theta}/\lambda\tau p\gamma$ from (A3) into (A4) to obtain

$$e = \frac{2\lambda w_0}{(a+\mu)c(\tau p)^{\theta}}, \tag{A5}$$

which is identical to equation (18) in the main text. Substituting (A5) into (A3) delivers

$$l = \left[\frac{\tau p\gamma(a+\mu)}{2w_0}\right]^2 z, \tag{A6}$$

which is identical to equation (17) in the main text.

Finally, substituting (A5) into (A1), where $e_i = e$, yields

$$w = w_0\left[1 + \frac{2\lambda^2 w_0}{(a+\mu)^2 c(\tau p)^{\theta}}\right], \tag{A7}$$

which is identical to the expected wage in the performance-pay sector $E(w_i)$, as indicated by (19). Under risk neutrality, linear contracts are optimal. They generate the same equilibrium values of effort e, the measure of workers hired l, and the (expected) wage in the performance-pay sector $E(w_i)$, as in the case of the same information among each entrepreneur and workers.

A.2 International trade

Let m denote the performance-pay-good quantity imported and $\tau = (1+t) \geq 1$ the tariff factor, where t is the ad-valorem tariff. The relative domestic price of the imported good is τp, where p is its world relative price. The aggregate tariff revenue is $R = (\tau - 1)pm$, and the per-capita tariff revenue is

$$r = \frac{R}{\overline{\overline{N}}} = \frac{(\tau-1)pm}{\overline{\overline{N}}}. \tag{A8}$$

The quantity imported is

$$m = Q^D - Q^S, \tag{A9}$$

where Q^D and Q^S are the aggregate quantities demanded and supplied.

The first step is to calculate the aggregate demand Q^D. Three distinct income groups consume the performance-pay good: workers in the traditional sector, workers in the performance-pay industry, and entrepreneurs. Consider first a worker in the traditional sector who receives disposable income I_0 equal to

$$I_0 = w_0 + r. \tag{A10}$$

The demand for the performance-pay good by workers in the traditional sector is

$$Q_0^D = N_0 \frac{\theta(w_0 + r)}{\tau p} = \frac{S_0 \theta(w_0 + r)}{w_0^{1/\delta} \tau p}, \tag{A11}$$

where $N_0 = S_0/w_0^{1/\delta}$ is the measure of individuals employed in the traditional sector.

In the performance-pay sector worker i earns disposable income $I_i = w_i + r$, where $w_i = b + \beta x_i$ exhibits the same distributional properties as x_i. Consequently, aggregate worker income equals $\int_0^{N_L} (w_i + r)\, di = N_L E(w_i) + N_L r$, where N_L is the measure of workers in the performance-pay sector, given by (33), the expected wage $E(w_i)$ is provided by (14), and the rebate r by (A8). As a result, the demand for the performance-pay good by all workers in this sector is

$$Q_L^D = N_L \frac{\theta}{\tau p} \left[w_0 + \frac{(\lambda\beta)^2}{2c(\tau p)^\theta} + r \right]. \tag{A12}$$

Each active entrepreneur receives income I_E equal to firm profit $\pi_z = w_0 z/\widehat{z}$ plus r:

$$I_E = \frac{w_0 z}{\widehat{z}} + r. \tag{A13}$$

The demand for the performance-pay good by all entrepreneurs is

$$Q_E = \frac{\theta}{\tau p} \left(\overline{N} \int_{\widehat{z}}^{\infty} I_E(z) g(z) dz \right) = \frac{\theta k w_0 N_E}{\tau p(k-1)} + \frac{\theta r N_E}{\tau p}, \tag{A14}$$

where $N_E = \overline{N}\widehat{z}^{-k}$ is the measure of active entrepreneurs, which is given by (30).

Consequently, the aggregate demand for the performance-pay good is

$$Q^D = Q_0^D + Q_L^D + Q_E^D = \frac{\theta}{\tau p} \left[w_0(N_0 + N_L + \frac{kN_E}{k-1}) + \frac{(\lambda\beta)^2}{2c(\tau p)^\theta} N_L + r\overline{N} \right], \tag{A15}$$

where the full employment of labor condition $\overline{N} = N_0 + N_L + N_E$ was used.

Substituting $N_0 = S_0/w_0^{1/\delta}$, N_L from (33), $N_E = \overline{N}\widehat{z}^{-k}$, and r from (A8) yields

$$Q^D = \frac{\theta}{\tau p} \left\{ \frac{S_0}{(w_0)^{(1-\delta)/\delta}} + \frac{w_0 k\overline{N}}{(k-1)} \left[\frac{\tau p \gamma(a+\mu)}{2w_0} \right]^2 \frac{1}{\widehat{z}^{(k-1)}} + \frac{w_0 k\overline{N}}{(k-1)} \frac{1}{\widehat{z}^k} + \frac{k\overline{N}}{(k-1)} \frac{(\lambda\tau p\gamma)^2}{2c(\tau p)^\theta} \frac{1}{\widehat{z}^{(k-1)}} \right\} + \frac{\theta(\tau-1)m}{\tau}. \tag{A16}$$

The second step is to calculate the aggregate supply Q^S. Firm output is $y_z = \gamma z^{1/2} l^{1/2} (a+$

$\mu+\lambda e$), where the equilibrium worker effort is given by

$$e = \frac{2\lambda w_0}{(a+\mu)c(\tau p)^\theta}. \tag{A17}$$

Substituting effort e from (A17) and the measure of employed workers l from (17) into the expression for firm output y_z yields

$$y_z = \frac{2w_0}{\tau p}\frac{z}{\widehat{z}}, \tag{A18}$$

where equation (27) was used. Integrating (A18) over all firms delivers the aggregate supply of the performance-pay good

$$Q^S = \int_{\widehat{z}}^{\infty} \overline{N} y(z) g(z) dz = \frac{2k\overline{N}}{\tau p(k-1)}\frac{w_0}{\widehat{z}^k}, \tag{A19}$$

where equation (27) was used again.

Inserting the expressions for aggregate demand Q^D from (A16) and aggregate supply Q^S from (A19) into (A9) yields

$$m = \frac{1}{[\theta+(1-\theta)\tau]p}\left[\frac{\theta S_0}{w_0^{(1-\delta)/\delta}} - \frac{(2-\theta)k\overline{N}w_0}{(k-1)\widehat{z}^k} + \frac{\theta k\overline{N}w_0}{(k-1)}\left[\frac{\tau p\gamma(a+\mu)}{2w_0}\right]^2\frac{1}{\widehat{z}^{(k-1)}} + \frac{\theta k\overline{N}}{(k-1)}\frac{(\lambda\tau p\gamma)^2}{2c(\tau p)^\theta}\frac{1}{\widehat{z}^{(k-1)}}\right]. \tag{A20}$$

Substituting $S_0/w_0^{1/\delta} = N_0$ and using the full employment condition $\overline{N} = N_0 + N_L + N_E$ delivers

$$m = \frac{w_0\overline{N}}{[\theta+(1-\theta)\tau]p}\left\{\theta(1-\frac{1}{\widehat{z}^k}) + \frac{\theta k}{(k-1)}\frac{\lambda^2}{\left[\lambda^2+\frac{c(\tau p)^\theta(a+\mu)^2}{2w_0}\right]}\frac{1}{\widehat{z}^k} - \frac{(2-\theta)k}{(k-1)}\frac{1}{\widehat{z}^k}\right\}, \tag{A21}$$

which is identical to equation (36) in the main text.

A.3 Equilibrium and computational algorithm

Table 1 presents the model's primitive exogenous parameters. Given these parameters, the general equilibrium solution to the model consists of the reservation wage's w_0 value satisfying equation (34). The following conditions determine this equilibrium. Everyone is fully employed and maximizes their expected utility. In addition, each of the two good markets clears. Finally, each entrepreneur maximizes firm profits by selecting the base payment b and bonus factor β through backward induction (consistent with a subgame perfect equilibrium).

All other endogenous variables, such as worker effort, bonus factor, base payment, firm profit, the measure of entrepreneurs, the performance pay incidence, the quantities of imports and exports, and the tariff revenue, can be calculated using the equilibrium value of the reservation wage.

The model's equilibrium is consistent with three schemes of personal income distribution. First, the personal real income distribution uses everyone's expected utility. This scheme considers the utility cost of worker effort and implies that all workers receive the same real income thanks to perfect mobility across the two sectors. The real income of active entrepreneurs rises linearly with talent. Second, the ex-ante personal income distribution employs the expected income of each abstracting from unobserved worker effort. Third, the ex-post personal income distribution employs realized (as opposed to expected) worker income and considers the implication of idiosyncratic uncertainty associated with worker output, resulting in compensation differences across workers within each firm in the performance-pay sector.

As stated, we chose the ex-post income distribution to calibrate the model because it fits the observed US income distribution better. The following algorithm is used to compute ex-post incomes.

1. Draw a random sample of $\overline{N} = 500,000$ individuals, with each individual assigned a talent level z according to the Pareto distribution defined by (29).

2. Treat the reservation wage w_0 as given, and solve for the cutoff talent level $\widehat{z}$ employing (27). The cutoff talent $\widehat{z}$ can be used to determine the number of entrepreneurs in the sample N_E.

3. For each entrepreneur with talent z, calculate the number of hired workers using the integer value of l determined by (17). For each hired worker, calculate her output using (7), where the equilibrium level of effort is given by (18). The idiosyncratic output component ξ_i is drawn from a log-normal distribution with zero mean and variance s^2, that is, $\ln(\xi_i) \sim N(0, s^2)$.

4. Calculate each hired worker's compensation $w_i(z)$ using the piece rate formula (8), where the bonus β is given by (16) and the base payment b by (11).

5. The market income of an entrepreneur with talent z who has hired $l(z)$ workers is determined as follows: firm revenue is given by $R(z) = p\tau y = p\tau\gamma x z^{1/2}[l(z)]^{1/2}$, where x is the average output of hired workers; the income of the entrepreneur is equal to ex-post firm profit defined as $\pi(z) = R(z) - W(z)$, where $W(z) = \sum_i w_i(z)$ is the total ex-post labor cost.

6. The ex-post income of an entrepreneur with talent z equals $\pi(z) + r$. The ex-post income of a worker hired by an entrepreneur with talent z equals $w_i(z) + r$, where the per-capita tariff revenue is determined by (35) and (36).

7. Follow this process for all entrepreneurs N_E (individuals with talent $z \geq \widehat{z}$). Assign ex-post incomes to each entrepreneur and worker in the performance-pay sector. Calculate the ex-post employment in the performance-pay sector N_L by summing up the workers hired by all entrepreneurs. Use the full employment condition to calculate the number of individuals in the traditional sector $N_0 = \overline{N} - N$.

8. Equation (34) determines the equilibrium value of the reservation wage w_0. If it is not satisfied, repeat the process by choosing different values of w_0 until this equation holds.

A.4 Sensitivity analysis

A.4.1 Normally-distributed output

In this subsection, we perform the same counterfactuals as in the main text but under the assumption of normally-distributed errors ξ_i. In practice, we do so by using the same parameters as summarized in Table 1, with one main modification: We now have to explicitly choose the standard deviation of ξ_i, σ, in addition to parameter μ and we select it such that the model again matches the Gini Coefficient of income, which is the case when σ equals 1.813.

We find that the findings we document in Section 4 largely align with those predicted by the baseline model. As evident from the tables below, many outcomes, such as the reservation wage or the size of each sector, do, in fact, only depend on the expected value of ξ_i and are therefore robust to mean-preserving changes in the underlying distribution. The main outcomes generally affected by distributional assumptions are the measures of inequality. We do, however, find that the qualitative predictions regarding these measures are similar when we use a normal distribution instead, and the results are also quantitatively comparable in most cases.

First, we find that greater trade protection still generates moderate increases in inequality, with an increase of nearly 2 percentage points compared to 2.81 in the baseline case. An increase in the Pareto shape parameter k continues to lower inequality, measured by the top one percent income share. An increase in σ increases inequality, especially when measured by the Gini Coefficient. Lastly, an increase in a lowers the Gini Coefficient by 2.05 percent which exactly matches the decline in the baseline case.

When ξ_i is normally distributed, the top one percent income share increases, while it declined in the baseline case. This result is mainly due to the symmetry of the normal distribution, generating fewer workers with very high productivity draws. Increases in a therefore, on average, benefit firms disproportionately more than in the log-normal case, with the result that the top one percent income share rises. Note that for higher values of σ, the results for the normal case are qualitatively the same as those we found in the log-normal case.

Table A1: The Effect of Trade Protection - Normally-distributed ξ_i

Endogenous Variable	$\tau = 3\%$ (1)	$\tau = 20\%$ (2)	Δ% (3)
Top 1% income share	8.17	8.19	0.27
Income Gini Coefficient	41.00	41.76	1.85
Performance pay incidence (in %)	45.00	48.15	7.00
Worker bonus factor	0.30	0.34	14.60
Reservation wage	0.46	0.52	14.60
Share - traditional sector (in %)	24.04	18.76	-21.95
Equilibrium effort level	0.95	0.95	-0.58
Equilibrium cutoff talent	1.75	1.69	-3.11
Imports	0.22	0.16	-28.11

Variable *Share - traditional sector* describes the fraction of workers that are employed in the traditional sector. All numbers are rounded to two decimal places.

Table A2: Comparative Statics: Changes in k - Normally-distributed ξ_i

Endogenous Variable	$k = 2.1$	$k = 2.5$	Δ%
Top 1% income share	8.17	5.81	-28.87
Income Gini Coefficient	41.00	37.58	-8.34
Performance pay incidence (in %)	45.00	41.22	-8.40
Worker bonus factor	0.30	0.28	-5.67
Reservation wage	0.46	0.43	-5.67
Share - traditional sector (in %)	24.04	26.73	11.20
Equilibrium effort level	0.95	0.90	-5.67
Equilibrium cutoff talent	1.75	1.58	-9.80
Imports	0.22	0.26	16.17

All values are computed based on the parameter values described in Table 1. The variable *Share - traditional sector* describes the fraction of workers that are employed in the traditional sector. All numbers are rounded to two decimal places.

Table A3: Comparative Statics: Changes in a - Normally-distributed ξ_i

Endogenous Variable	$a = 1.22$	$a = 1.5$	$\Delta\%$
Top 1% income share	8.17	8.29	1.45
Income Gini Coefficient	41.00	40.18	-2.05
Performance pay incidence (in %)	45.00	47.15	4.77
Worker bonus factor	0.30	0.29	-1.72
Reservation wage	0.46	0.49	7.21
Share - traditional sector (in %)	24.04	21.18	-11.89
Equilibrium effort level	0.95	0.94	-1.72
Equilibrium cutoff talent	1.75	1.73	-1.07
Imports	0.22	0.19	-13.12

All values are computed based on the parameter values described in Table 1. The variable *Share - traditional sector* describes the fraction of workers that are employed in the traditional sector. All numbers are rounded to two decimal places.

Table A4: Comparative Statics: Changes in σ - Normally-distributed ξ_i

Endogenous Variable	$\sigma = 1.8$	$\sigma = 2.5$	$\Delta\%$
Top 1% income share	8.17	8.33	1.96
Income Gini Coefficient	41.00	51.25	25.00
Performance pay incidence (in %)	45.00	45.00	0.00
Worker bonus factor	0.30	0.30	0.00
Reservation wage	0.46	0.46	0.00
Share - traditional sector (in %)	24.04	24.04	0.00
Equilibrium effort level	0.95	0.95	0.00
Equilibrium cutoff talent	1.75	1.75	0.00
Imports	0.22	0.22	0.00

All values are computed based on the parameter values described in Table 1. The variable *Share - traditional sector* describes the fraction of workers that are employed in the traditional sector. All numbers are rounded to two decimal places.

Table A5: The Effect of Trade Protection - No-effort model

Endogenous Variable	$\tau = 3\%$ (1)	$\tau = 20\%$ (2)	$\Delta\%$ (3)
Top 1% income share	10.94	11.71	7.04
Income Gini Coefficient	48.38	49.99	3.32
Performance pay incidence (in %)	47.24	51.18	8.34
Worker bonus factor	0.00	0.00	0.00
Reservation wage	0.42	0.48	14.30
Share - traditional sector (in %)	27.87	21.86	-21.58
Equilibrium effort level	0.00	0.00	0.00
Equilibrium cutoff talent	1.94	1.87	-3.74
Imports	0.29	0.22	-23.08

Variable *Share - traditional sector* describes the fraction of workers that are employed in the traditional sector.

A.4.2 Tariffs in the no-effort model

We perform the same counterfactuals as in the main text but under the assumption that no costly effort is being exerted. In practice, we do so by setting parameter $\lambda = 0$ and the bonus factor $\beta = 0$. This change results in zero equilibrium costly effort level ($e = 0$), both in the factual and the counterfactual cases. Regarding the counterfactual changes we studied in Section 4, we find that the no-effort model's predictions differ from those in the baseline case, especially regarding the implications for trade and as evident from Tables 3 and A5.

First, the model predicts that the impact of a tariff on imports will be smaller than in the baseline case. The economy imports more because its workers are less productive. Consequently, the quantity imported falls from 29 percent to 22 percent. Second, the degree of and change in inequality are more significant than the baseline case. These results rely on two effects. By exerting minimum effort ($e = 0$) and receiving no bonus, workers earn lower wages, resulting in smaller earnings on the lower tail of the income distribution. The wage reduction is more significant than the firm profit fall, resulting in a higher top one percent income share. Tables 3 and A5. reflect this reasoning by showing that the top one percent income share increases by 7 percent compared to 2 percent in the baseline model. Performance-pay compensation significantly mitigates the impact of tariffs on top incomes, reducing the effect by more than half.

References

[1] Edmans, Alex, and Xavier Gabaix, "Executive Compensation: A Modern Primer," *Journal of Economic Literature*, 2016, 54(4), 1232-1287.

[2] Holmstrom, Bengt, and Paul Milgrom, "Aggregation and Linearity in the Provision of Intertemporal Incentives," *Econometrica*, 1987, 55(2), 303-328.